

Uses of a small field value which falls from a metastable maximum over cosmological times

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Abstract

We consider a small, metastable maximum vacuum expectation value b_0 of order of a few eV, for a pseudoscalar Goldstone-like field, which is related to the scalar inflaton field ϕ in an idealized model of a cosmological, spontaneously-broken chiral symmetry. The b field allows for relating semi-quantitatively three distinct quantities in a cosmological context.

- (1) A very small, residual vacuum energy density or effective cosmological constant of $\sim \lambda b_0^4 \sim 2.7 \times 10^{-47} \text{ GeV}^4$, for $\lambda \sim 3 \times 10^{-14}$, the same as an empirical inflaton self-coupling.
- (2) A tiny neutrino mass, less than b_0 .
- (3) A possible small variation downward of the proton to electron mass ratio over cosmological time. The latter arises from the motion downward of the b field over cosmological time, toward a nonzero value. Such behavior is consistent with an equation of motion.

We argue that hypothetical b quanta, potentially inducing new long-range forces, are absent, because of negative, effective squared mass in an equation of motion for b -field fluctuations. The assumed flatness of a potential maximum involves a small inverse-time parameter $\mu \ll 1/t_0$, where t_0 is the present age of the universe.

There exist a number of essential physical quantities which probably have a cosmological origin, and which may have a time variation over cosmological time scales, that is time scales of the order of the present age of the universe, or much

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longer. Such quantities are: (1) a small, effective cosmological constant, (2) very small neutrino mass, and (3) a possible small time variation of the proton to electron mass ratio (m_p/m_e) [1, 2]. In this paper, we outline the essential elements of a model which attempts to relate conceptually, and in a numerically approximate manner, the above phenomena. The model utilizes parameters which characterize initial inflation, that is a scalar inflaton field ϕ which falls from a potential maximum, at or a little above the Planck mass M_{Pl} , to a minimum denoted here by ϕ_0 , just below M_{Pl} . The ϕ self-coupling is characterized by an empirically [3], very small, dimensionless coupling $\lambda \sim 3 \times 10^{-14}$, and the time scale is of order $(1/\sqrt{\lambda}\phi_0) \sim 2.4 \times 10^{-36}$ s (for $\phi_0 \sim 10^{18}$ GeV).^{F1} The latter corresponds to an inflaton mass of order $\sqrt{\lambda}\phi_0 \sim 2 \times 10^{11}$ GeV. We have estimated [4, 5, 6, 7] that (metastable) inflaton quanta could constitute a significant contribution to cold dark matter in the present epoch (extremely massive, essentially non-interacting particles). We have illustrated [4, 5] how a maximum, and a minimum, of an effective, ϕ potential-energy density, can arise from certain kinds of radiative corrections to a $\lambda\phi^4$ potential-energy density. Inflation occurs with the inflaton field at the maximum and during its movement to the minimum. The cosmological chiral model [4, 5, 6, 7], spontaneously broken by ϕ_0 , contains a Goldstone-like [8] pseudoscalar field b , so that the self-coupling is $\lambda(\phi^2 + b^2)^2$. In addition, there is a massive, primordial neutrino-like neutral lepton L , with Lagrangian couplings

$$-g_L(\bar{L}L\phi + i\bar{L}\gamma_5 Lb) \quad (1)$$

These terms and the self-coupling are initially invariant under the chiral transformations [9] (for infinitesimal β)

$$\begin{aligned} \phi &\rightarrow \phi - \beta b, & b &\rightarrow b + \beta\phi \\ L &\rightarrow L - \frac{i\beta}{2}\gamma_5 L, & \bar{L} &\rightarrow \bar{L} - \frac{i\beta}{2}\bar{L}\gamma_5 \end{aligned} \quad (2)$$

An essential relationship is contained in the presence of the b self-coupling λb^4 , with the same coupling parameter as for the inflaton. This gives a residual ^{F2} vacuum energy density of $\lambda b_0^4 \sim 2.7 \times 10^{-47}$ GeV⁴, if the b field has a nonzero vacuum expectation value $b_0 \sim 5.5$ eV. A nonzero vacuum expectation value for a pseudoscalar field, spontaneously breaks CP invariance [10, 11], in a cosmological context ^{F3}. An attempt is made to make an independent estimate of b_0 [6, 7], by coupling the b field to a neutrino, with Lagrangian terms

$$-\mathcal{L}_\nu = \tilde{m}_\nu \bar{\nu}\nu + ig_\nu \bar{\nu}\gamma_5 \nu b \quad (3)$$

^{F1}As counted from the time of ϕ leaving its value $\gtrsim M_{Pl}$ at the effective potential maximum.

^{F2}Residual means after normalizing the minimum in the potential-energy density of the inflaton field to zero.

^{F3}Spontaneous CP violation is a motivation for considering a nonzero vacuum expectation value for the b field. For a brief time near to $\sim 10^{-36}$ s, there can be a CP-violating effect such as an antineutrino-neutrino asymmetry from a primary, radiation-producing decay process.[6, 7]

With the unitary transformation

$$\nu \rightarrow e^{-i\gamma_5\alpha_\nu/2}\nu, \quad \bar{\nu} \rightarrow \bar{\nu}e^{-i\gamma_5\alpha_\nu/2} \quad (4)$$

for $\tan \alpha_\nu = g_\nu b_0/\tilde{m}_\nu$, $-\mathcal{L}_\nu$ becomes ^{F4}

$$-\mathcal{L}_\nu = m_\nu \bar{\nu}\nu + g_\nu(i \cos \alpha_\nu \bar{\nu}\gamma_5\nu + \sin \alpha_\nu \bar{\nu}\nu)(\delta b) \quad (5)$$

with $b \rightarrow b_0 + \delta b$, $m_\nu = \sqrt{\tilde{m}_\nu^2 + (g_\nu b_0)^2}$, $\cos \alpha_\nu = \tilde{m}_\nu/m_\nu$, $\sin \alpha_\nu = g_\nu b_0/m_\nu$. Consider the situation if $\tilde{m}_\nu \ll (g_\nu b_0)$ (essentially, vanishing “bare” neutrino mass \tilde{m}_ν). Then, $\cos \alpha_\nu \sim 0$, $\sin \alpha_\nu \sim 1$; the effective mass of the neutrino is $\sim (g_\nu b_0)$, numerically ~ 0.05 eV for ^{F5} $g_\nu \sim 0.01$. Thus, an explicit breaking of the initial chiral symmetry which is associated with neutrino mass in Eqs. (3,5), is of the order of the breaking arising from the spontaneously-broken CP invariance ($b_0 \neq 0$). The third of our hypothetical connections, concerns an empirically possible time variation, in particular downward [1, 2], of (m_p/m_e) . In analogy, with the above argument for neutrino mass, a hypothetical coupling of the b field to primordial, ordinary quarks gives a quark mass contribution of $g_b b_0$; subsequently for three confined valence quarks, a nucleon mass contribution of order $3g_b b_0$. Here, we have assumed that primordial quarks have zero intrinsic (“bare”) mass. We assume that electroweak symmetry-breaking generates the standard-model MeV mass contribution for light quarks at a later time, and we assume that this contribution simply adds to the primordial mass contribution estimated here, $g_b b_0$. This is the assumption that the electroweak mass term arises from the Higgs vacuum expectation value times a tiny coupling to the quark field which has acquired a small mass term $g_b b_0$.

Here, we concentrate upon giving an illustrative argument which suggests the consistency of the possibility for the above third connection. The essential physical assumption is that the b field can fall over cosmological time intervals, from a maximum value b_0 , to a nonzero value, say $\sqrt{\epsilon}b_0$ ($0 < \epsilon < 1$). An effective vacuum energy density is constructed (below) for b , which takes as a constant term λb_0^4 with λ the tiny self-coupling of the inflaton field. This is suggested by the cosmological chiral model outlined in the introduction. In our example below, we will consider ϵ to be a little less than unity. If the dimensionless parameter ϵ is not identically 0 (as usually is assumed), a value near unity would be, a priori, reasonable (note also the relevance of $\epsilon \lesssim 1$ to the discussion in appendix III). A second assumption is that potentially new, long-range forces due to exchange of b quanta do not occur, because of negative, effective squared mass in an equation of

^{F4}CP noninvariance is formally manifest in the scalar coupling of (δb) , with reference to the pseudoscalar coupling in (1) (and in (5)).

^{F5}In the numerical, renormalization-group calculations [4, 5] which give the inflaton potential a maximum and a minimum near to the Planck mass, we found a representative coupling parameter of ϕ (and b) to a primordial, neutrino-like massive lepton L , to be $g_L \sim 0.01$.

motion for b -field fluctuations. It is assumed that quanta with negative squared mass are not present (i. e. superluminal tachyons [12, 13, 14, 15]).

A usual equation of motion is

$$\ddot{b} + 3H\dot{b} + V'(b) = 0, \quad V'(b) = dV(b)/db \quad (6)$$

with $V'(b)$ the derivative of an effective vacuum energy density. For simplicity in the present argument, we approximate the time-dependent Hubble parameter over long cosmological times, approximating the present epoch, as functionally,

$$H(t) \sim \frac{2}{3t} f(\Omega_{vac}(t_0)) \sim 1/t \quad (7)$$

for $f(\Omega_{vac}(t_0)) \sim 1.5$, where $f(\Omega_{vac}(t)) = \Omega_{vac}^{-1/2} \ln((1 + \Omega_{vac}^{1/2})/(1 - \Omega_{vac}^{1/2}))$, $\Omega_{vac}(t) = \rho_{vac}/\rho_C(t)$. The vacuum and critical energy densities are ρ_{vac} and $\rho_C(t)$ respectively, with $\Omega_{vac}(t_0)$ assumed to be ~ 0.7 at the age t_0 of the present epoch. The factor $f > 1$ reflects the fact that the present age is somewhat greater than $\sim 2/3H(t_0)$, because of the assumed presence of Ω_{vac} in an assumed flat universe [16]. First, consider that $V'(b)$ can be neglected in the equation of motion (6). We then have approximately

$$\ddot{b} + 3\frac{\dot{b}}{t} = 0 \quad (8)$$

For this equation, an approximate solution for long cosmological times, greater than some scale \bar{t} which is significantly less than t_0 , is

$$b(t) \sim constant + \frac{(1 - \sqrt{\epsilon})b_0}{2} \frac{1}{(t/\bar{t})^2} \quad (9)$$

as is verified by differentiation in (8).^{F6} For epochs less than \bar{t} , b is assumed to be nearly constant at a value b_0 . Now, consider inclusion of a relatively small $V'(b)$ given, as an explicit example, by the following form which is assumed to hold in the domain $\sqrt{\epsilon}b_0 \leq b \leq b_0$, with ϵ a little less than unity. This involves a small (through μ^2), explicit chiral-symmetry breaking.

$$V'(b) = \mu^2 \left[b_0 - \frac{b_0^2}{b} \left\{ \frac{(b^2 - \epsilon b_0^2)}{(1 - \epsilon)b_0^2} \right\}^2 \right] \quad (10)$$

Here, μ is a fixed parameter with dimension of inverse time, assumed to be set by a time scale considerably greater than the present epoch t_0 . Therefore, the V' term in (6) is relatively small, at least out to times much greater than t_0 . In (10), we have $V'(b_0) = 0$, and $V'(b) > 0$ as $b \rightarrow \sqrt{\epsilon}b_0$. From (12) below, the latter value corresponds to the zero of $V''(b)$, i. e. the point of change from negative to

^{F6}The essence of the physical argument is not changed if the fall at large times is stronger, say exponential.

positive values. We do not extrapolate the form in (10) for b below this point. In particular with ϵ near zero, such extrapolation of the form (10) has a zero near $b = \epsilon^2 b_0$, with $V'/\mu^2 b_0$ becoming strongly negative at still lower b , and with V''/μ^2 ((12) below) strongly positive. The physical assumption which underlies the speculation in this paper is that a $b < \sqrt{\epsilon} b_0$ where a $V'(b)$ is zero, is reached only at times much greater than t_0 (and thus than the \bar{t} in (9)), i. e. in epochs characterized by μ^{-1} . The usual, effective equation of motion for free propagation of potential quanta of the b field, denoted by δb , is

$$\square(\delta b) + V''(b)(\delta b) = 0 \quad (11)$$

where $V'' = dV'/db$ gives the effective, squared mass. Evaluating this from (10), gives

$$V''(b) = \mu^2 \left[\frac{b_0^2}{b^2} \left\{ \frac{(b^2 - \epsilon b_0^2)}{(1 - \epsilon)b_0^2} \right\}^2 - 4b_0^2 \frac{(b^2 - \epsilon b_0^2)}{((1 - \epsilon)b_0^2)^2} \right] \quad (12)$$

Thus, the effective, squared mass is negative, $\mu^2(1 - 4/(1 - \epsilon))$ at $b = b_0$; it approaches zero as $b \rightarrow \sqrt{\epsilon} b_0$. The positive, very small values of $V'(b)$ and the negative V'' , imply that the effective cosmological constant from the energy density $\cong (\lambda b_0^4 + V'(\Delta b) + V''(\Delta b)^2/2)$, with $\Delta b < 0$, has a completely negligible downward movement. We emphasize that the first, constant term in this energy density, contains the tiny inflaton self-coupling λ , as suggested by the initial chiral symmetry. The energy scale is related to the very small scale of neutrino mass, of order b_0 . The extreme flatness (metastability over very long cosmological times) is associated with the parameter μ^2 in Eqs. (10,12). This is an essential assumption, which motivates the title of this paper.^{F7}

The example in the previous paragraph is intended only to illustrate the physical possibility of incorporating (3) together with the relations (1) and (2), as stated in the opening paragraph of this paper. There are predictions.

- (1) The effective cosmological constant has no discernable time variation, i. e. the residual vacuum energy density in the b field is, in effect, a constant.
- (2) There are, in principle, discernable, small time variations downward, in neutrino mass, and in (m_p/m_e) ^{F8}, as contributions from the b field go down with $b_0 \rightarrow \sqrt{\epsilon} b_0$.

^{F7} Assumed contributions of zero-point energies of standard quanta to the vacuum energy density are not considered in this paper. The presence in the model of a long time (length) scale ($1/\sqrt{\lambda} b_0$, note appendix I), might allow for the possibility that these are limited by a smaller value, of order $(\sqrt{\lambda} b_0)^4$.

^{F8} A large coupling parameter g_b , of b to quarks, is necessary to reach the suggested [1, 2] time variation of (m_p/m_e) , if attributed to a Δm_p . The electron mass can change, but if the leptonic coupling is like that estimated for neutrinos, $g_\nu \sim 0.01$, then the hypothetical downward change in (m_p/m_e) is probably controlled by the downward change in m_p .

- (3) Cold dark matter may well involve metastable quanta of the scalar inflaton field ϕ , that is super-massive particles at a mass scale of a few times 10^{11} GeV. Then, dark matter is intrinsically related to the existence of the scalar field whose vacuum energy density induces initial inflation. And the related b field is responsible for entrance at present into a period of inflation, induced by its vacuum energy density. Thus, in this model, there is an attempt to relate the different phenomena.

The new general idea is that there is a small energy scale b_0 associated with the early universe, in addition to the usual very high energy scales i. e. inflaton field ϕ_0 and radiation temperature. In the following appendices, we indicate some possibly suggestive, numerical properties of certain combinations of these scales.

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Appendix I

At the end of inflation, there are two very high, energy-scale parameters, $\phi_0 \cong 10^{18}$ GeV and the initial radiation temperature $T_0 \cong 10^{15.5}$ GeV. But there is also a very small, dimensionless parameter λ , of the order of 10^{-14} .^{F9} A hypothetical small energy scale $b_0 \cong 5.5$ eV is obtained from a conjectured relation $\lambda^2 = b_0/\phi_0 = 5.5 \times 10^{-27}$ (for $\lambda = 7.4 \times 10^{-14}$). There are then two time-scale parameters: $1/\sqrt{\lambda}\phi_0 = 2.4 \times 10^{-36}$ s at the end of inflation; and $1/\sqrt{\lambda}b_0 = 4.4 \times 10^{-10}$ s. Over the latter time scale, the radiation energy scale evolves downwards to a relatively low scale

$$T_0 \left(\frac{2.4 \times 10^{-36} \text{ s}}{4.4 \times 10^{-10} \text{ s}} \right)^{1/2} = h = 234 \text{ GeV} \quad (13)$$

This scale is almost the empirical Higgs energy scale (the assumed nonzero vacuum expectation value v) of the standard model. From this viewpoint, it is a time scale which determines h , i. e. the temperature at which a determined value for h is metastable. The coupling g_e , of $v/\sqrt{2}$ to electrons, is empirically another small number: $g_e^2 \cong 9 \times 10^{-12}$. Again, a ratio of energy scales almost gives this very small number $g^2 = b_0/h = 23.5 \times 10^{-12}$.^{F10} In summary, the above emphasis on

^{F9}Heuristically, this can be seen by equating the maximum energy density in the inflaton field, to the initial (maximum) radiation energy density, which appears just after inflation. Then, $\sim \lambda M_{\text{Pl}}^4 \sim T_0^4$ gives $\lambda \sim 10^{62} \text{ GeV}^4 / 10^{76} \text{ GeV}^4 = 10^{-14}$.

^{F10}A ratio of energy scales involving b_0 is close to $\alpha^2 = 5.3 \times 10^{-5}$, that is $b_0/m_e = 5.5 \text{ eV} / 0.5 \text{ MeV} = 1.1 \times 10^{-5}$.

the role of λ allows for a suggestive numerical hierarchy.

$$\begin{aligned} b_0 &= \lambda^2 \phi_0 \propto \lambda^2 \\ h &= \lambda \phi_0 (T_0/\phi_0) \propto \lambda \\ g^2 &= \lambda (\phi_0/T_0) \propto \lambda, \text{ so } g \propto \sqrt{\lambda} \end{aligned} \tag{14}$$

The essential parameters are ϕ_0 , near to M_{Pl} , and λ^2 (or alternatively b_0 , near to neutrino mass). Here, it is the extreme smallness of λ which relates the high energy scale ϕ_0 (or T_0) to the much lower energy scale h .

Appendix II - “Cosmic coincidence”

The present ratio of vacuum energy density ρ_Λ , to approximate dark-matter energy density $\rho_{\text{d.m.}}$, empirically $\rho_\Lambda/\rho_{\text{d.m.}} \sim 3$, appears to be a curious accident, in particular when these densities are assumed to originate in completely different dynamics, and the matter density falls with the expansion of the universe. Here, we note that the ratio of order unity is not particularly accidental, when the two densities are considered together as functions of only two quantities: the very large energy scale ϕ_0 , and the very small dimensionless parameter λ . We have^{F11}

$$\rho_\Lambda = \lambda b_0^4 = \lambda (\lambda^2 \phi_0)^4 = \lambda^9 \phi_0^4 \tag{15}$$

$$\begin{aligned} \rho_{\text{d.m.}} &= (\rho_{\text{d.m.}})_0 \left\{ \left(\frac{10^{-36} \text{ s}}{\bar{t}} \right)^{3/2} \left(\frac{\bar{t}}{10^{11} \text{ s}} \right)^{3/2} \left(\frac{10^{11} \text{ s}}{4 \times 10^{17} \text{ s}} \right)^2 \right\} \\ &= \left(\frac{1}{2} f(\lambda) m_\phi^4 \right) \{ (\lambda^4)^{3/2} \times (\sim \lambda^{0.39}) \} \\ &= (\lambda^{2.5} \phi_0^4) \{ (\lambda^6) \times (\sim \lambda^{0.39}) \} \\ &\cong \lambda^{8.89} \phi_0^4 \end{aligned} \tag{16}$$

In (16), $(\rho_{\text{d.m.}})_0$ is the initial energy density for inflatons created at $\sim 10^{-36}$ s. This density is written in terms of the inflaton mass $m_\phi \cong \sqrt{\lambda} \phi_0$, as $m_\phi^2/2 \times (f(\lambda) m_\phi^2)$ with an assumption for the fluctuation scale [4, 5], $\lambda < f(\lambda) < 1$.^{F12} As a definite example, we used in (16) the geometric mean $f(\lambda) = \sqrt{\lambda}$. The terms bracketed as $\{ \dots \}$ give the time evolution to the present age of the universe, $\sim 4 \times 10^{17}$ s. The first two terms give evolution to $\sim 10^{11}$ s, taken here as an approximate time of matter dominance. The second form of the first term on the r. h. s. explicitly uses a hypothetical λ dependence for setting the scale \bar{t} ; \bar{t} is determined by $\{(1/m_\phi) \times \bar{t}\}^{1/2} \sim 1/\sqrt{\lambda} b_0 \rightarrow \bar{t} \sim 10^{-36} \text{ s}/\lambda^4 \sim 3 \times 10^{16} \text{ s}$,

^{F11}For $\lambda \sim 7.4 \times 10^{-14}$, Eq. (15) gives ρ_Λ above the empirical value by only a factor of about 2.5.

^{F12}This is consistent with an estimate of the size of f from production of massive dark matter by a time-varying gravitational field, when H is of order m_ϕ . [6, 7]

using $b_0 = \lambda^2 \phi_0$, (this \bar{t} is about a characteristic time for galaxy-halo formation from dark matter). This conjecture equates a mean time scale - formed from the product of an initial, very short time scale $1/m_\phi \sim 1/\sqrt{\lambda}\phi_0 \sim 10^{-36}$ s (in which the field b_0 is established), with the long time scale \bar{t} , which characterizes subsequent, downward change in the b field - to an intrinsic time scale associated with the small b_0 . The last term gives evolution to the present. The last two terms are expressed as an effective, small power of λ . Eqs. (15,16) give^{F11}

$$\rho_\Lambda/\rho_{\text{d.m.}} \sim 1/30 \quad (17)$$

This differs from the approximate empirical value by only a factor of about 90. From this point of view, a ratio of order of unity may be explainable in terms of only two primary, dynamical quantities, ϕ_0 and λ^2 (or alternatively, b_0).

Appendix III: Succession of universes

The idea in this paper lends itself to a concept of an indefinitely large succession of universes, which are characterized by an ever-decreasing initial, and residual vacuum energy density; and by an ever-decreasing “initial” radiation and matter energy density (and thus subsequent matter energy density). Consider a universe (like ours) with an initial vacuum energy density in the inflaton field $\lambda\phi^4 \sim \lambda M_{\text{Pl}}^4$ (which immediately goes to radiation and matter, mainly inflaton mass $\sim \sqrt{\lambda}\phi_0$), and a residual vacuum energy density of order λb_0^4 (λ^2 given by b_0/ϕ_0 , $b_0 \neq 0$, $\phi_0 \lesssim M_{\text{Pl}}$). After an indefinitely long time, an indefinitely large space(-time) volume exists, with nonzero, residual vacuum energy density. Consider that in this large volume a “speck” occurs with inflaton field $\sim \sqrt{\epsilon}M_{\text{Pl}} \rightarrow \sqrt{\epsilon}\phi_0$ (so λ^2 is the same tiny dimensionless parameter given by the ratio of fields $\sqrt{\epsilon}b_0/\sqrt{\epsilon}\phi_0$). Assume that in the speck, a residual vacuum energy density is established at a maximum value $\lambda(\sqrt{\epsilon}b_0)^4$ ($\epsilon \lesssim 1$), corresponding to the initial field value $\sqrt{\epsilon}b_0$. This speck immediately inflates to a universe with a residual vacuum energy density reduced by a factor of $(\sqrt{\epsilon})^4 = \epsilon^2$ and with radiation and matter energy densities reduced by ϵ^2 . Repeating this argument N times, each time within an immediately preceding large space-time volume, gives rise to a universe in which initially, both residual vacuum energy density and radiation-matter energy density tend to zero, like $(\epsilon^2)^N$ i. e. an empty universe speck (embedded in an infinite space-time volume). Consider as a formal exercise with ϵ is a little less than unity, how many times (N_0) this would have had to occur in the past, in order to have produced effective, quartic self-coupling of the vacuum field values (i. e. ϕ_0, b_0 , as in our universe), which is of order unity. Then $\lambda/(\epsilon^2)^{N_0} \sim 1 \rightarrow N_0 \sim 1.6 \times 10^2$ for an ϵ of order 0.9. (ϕ_0 and b_0 would have increased by a factor of $10^{3.5}$. The initial radiation temperature T_0 , would be of order M_{Pl} . Alternatively, given an initial $T_0 \sim 10^{15.5}$ GeV in our universe, in order that an energy density in an earlier universe not exceed M_{Pl}^4 , λ must be as small as about

$(10^{-3.5})^4 = 10^{-14}$). Ours arises as one of the “livable” universes, in a succession of embedded universes with the same initial, dimensionless parameters λ and ϵ .^{F13}

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^{F13}In a certain sense, a (classical) direction of time is defined by a positive value of the pseudoscalar field b_0 , with the conjecture $\lambda^2 = b_0/\phi_0$. For $t \rightarrow -t, b_0 \rightarrow -b_0$, but $\lambda^2 \rightarrow -\lambda^2$ is not permissible.